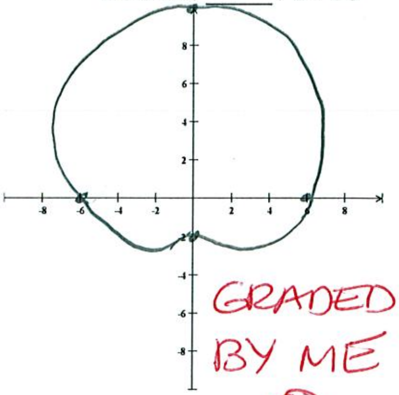
Consider the polar curve $r = -6 + 4\sin\theta$.

SCORE: ______ / 8 PTS



[a] Sketch the shape & position of the curve on the Cartesian graph paper on the right using the process shown in lecture and in the website handout.

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[b] State the full name of the shape of the curve.

LIMACON WITH DIMPLE 1

[c] Find the slope of the tangent line to the curve corresponding to the point $\theta = \frac{\pi}{6}$. Simplify your final answer, including rationalizing the denominator if applicable.

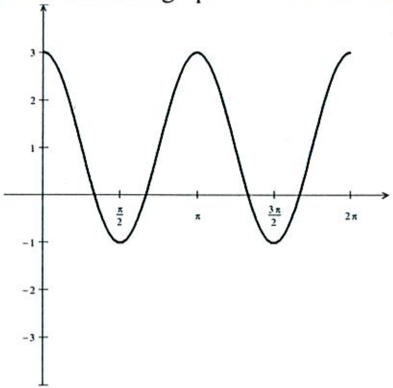
$$\frac{dy}{dx} = \frac{4\cos\theta\sin\theta + (-6+4\sin\theta)\cos\theta}{4\cos\theta\cos\theta - (-6+4\sin\theta)\sin\theta} \quad @\ \theta = \frac{\pi}{6}$$

1 1

$$= \frac{4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(-6+4\cdot\frac{1}{2}\right)\frac{\sqrt{3}}{2}}{4\left(\frac{\sqrt{3}}{2}\right)^2 - \left(-6+4\cdot\frac{1}{2}\right)\frac{1}{2}} = \frac{\sqrt{3}-2\sqrt{3}}{3+2} = \boxed{-\frac{\sqrt{3}}{5}}$$

1

The Cartesian graph $r = 1 + 2\cos 2\theta$ is shown below on the left. **SCORE: _____ / 7 PTS**



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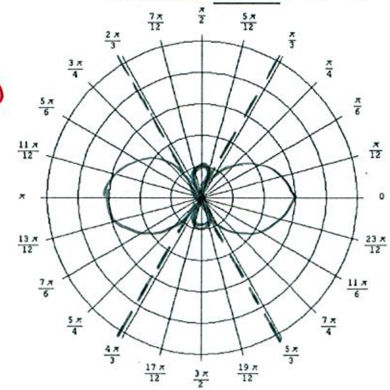

[a] Find **algebraically** all values of $\theta \in [0, 2\pi)$ at which the polar curve $r = 1 + 2\cos 2\theta$ goes through the pole. Show the logic that leads to the answers. (No credit for guess & check.)

$\frac{1}{2}$ $1 + 2\cos 2\theta = 0$

$\cos 2\theta = -\frac{1}{2}$

1 $2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ 1 $\theta \in [0, 2\pi) \longrightarrow 2\theta \in [0, 4\pi)$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

1 1

[b] By inspecting the Cartesian graph, find all values of $\theta \in [0, 2\pi)$ at which the polar curve changes from moving away from the pole to moving towards the pole.

$\theta = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ $\frac{1}{2}$

[c] Using the answers to [a] and [b], sketch the shape & position of the polar curve $r = 1 + 2\cos 2\theta$ on the polar graph paper above.

Suppose you are given the polar curve $r = f(\theta)$.

SCORE: _____ / 3 PTS

[a] How are the graphs of the polar curves $r = f(\theta)$ and $r = f(\theta + \frac{\pi}{4})$ related to each other ?

1

$r = f(\theta + \frac{\pi}{4})$ IS $r = f(\theta)$ ROTATED $\frac{\pi}{4}$ CLOCKWISE AROUND POLE

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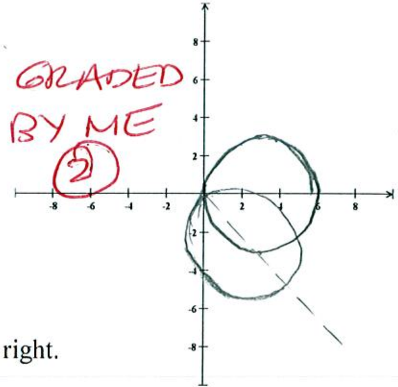


[b] Sketch the polar curves $r = 6\cos\theta$ and $r = 6\cos(\theta + \frac{\pi}{4})$ on the Cartesian graph paper on the right.

The polar curves $r = \cos(\theta) + 2$ and $r = \sin(\theta) - 2$ intersect at two points. **SCORE: _____ / 5 PTS**

Find **algebraically** the polar coordinates of those two points. Show the logic that leads to the answers. (No credit for guess & check.)

1 $\boxed{\begin{array}{l} \cos(\theta) + 2 = 0 \\ \cos\theta = -2 \quad \text{IMPOSSIBLE} \end{array}}$

$\cos\theta + 2 = \sin\theta - 2$

$\frac{1}{2}$ $\boxed{\begin{array}{l} \cos\theta = \underbrace{\sin\theta - 4}_{\in[-1-4,\,1-4]=[-5,-3]} \\ \text{IMPOSSIBLE} \end{array}}$

$-r = \cos(\pi+\theta) + 2$ OR $-r = \sin(\pi+\theta) - 2$

$-r = -\cos\theta + 2$ OR $-r = -\sin\theta - 2$

1 $\boxed{r = \cos\theta - 2}$ OR $\rightarrow$ $r = \sin\theta + 2$

1 $\boxed{\cos\theta - 2 = \sin\theta - 2}$ OR $\rightarrow$ $\cos\theta + 2 = \sin\theta + 2$

$\cos\theta = \sin\theta$

$1 = \tan\theta$

1 $\boxed{\theta = \frac{\pi}{4}, \frac{5\pi}{4}}$

$\frac{1}{2}$ $\boxed{\left(\frac{\sqrt{2}}{2} - 2, \frac{\pi}{4}\right)}$ AND $\boxed{\left(-\frac{\sqrt{2}}{2} - 2, \frac{5\pi}{4}\right)}$ $\frac{1}{2}$

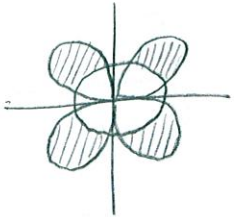
$\downarrow$ OR $\qquad \downarrow$ OR

$\left(2 - \frac{\sqrt{2}}{2}, \frac{5\pi}{4}\right) \qquad \left(\frac{\sqrt{2}}{2} + 2, \frac{\pi}{4}\right)$

Consider the polar curves $r = 2\sin 2\theta$ and $r = 1$. ★ GRADE ONLY AGAINST 1 SOLUTION FOR EACH PART

SCORE: _____ / 7 PTS

[a] Write, **but do NOT evaluate**, an integral (or sum of integrals) for the area that lies inside $r = 2\sin 2\theta$ but outside $r = 1$.



$2\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$ IN Q_1

$$\underbrace{4}_{\frac{1}{2}} \cdot \underbrace{\frac{1}{2}}_{\frac{1}{2}} \underbrace{\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}}}_{1} \underbrace{(4\sin^2 2\theta - 1)}_{1}\, d\theta$$

OR

$$\underbrace{8}_{\frac{1}{2}} \cdot \underbrace{\frac{1}{2}}_{\frac{1}{2}} \underbrace{\int_{\frac{\pi}{12}}^{\frac{\pi}{4}}}_{1} \underbrace{(4\sin^2 2\theta - 1)}_{1}\, d\theta$$

[b] Write, **but do NOT evaluate**, an integral (or sum of integrals) for the area that lies inside both $r = 2\sin 2\theta$ and $r = 1$, ie. the area where the insides of the two curves overlap.

$$\underbrace{4}_{\frac{1}{2}} \cdot \underbrace{\frac{1}{2}}_{\frac{1}{2}} \left[\underbrace{\int_0^{\frac{\pi}{12}} (4\sin^2 2\theta)\, d\theta}_{1} + \underbrace{\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 1^2\, d\theta}_{1} + \underbrace{\int_{\frac{5\pi}{12}}^{\frac{\pi}{2}} (4\sin^2 2\theta)\, d\theta}_{1} \right]$$

OR

$$\underbrace{8}_{1} \cdot \underbrace{\frac{1}{2}}_{\frac{1}{2}} \left[\underbrace{\int_0^{\frac{\pi}{12}} (4\sin^2 2\theta)\, d\theta}_{1\frac{1}{2}} + \underbrace{\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 1^2\, d\theta}_{1} \right]$$